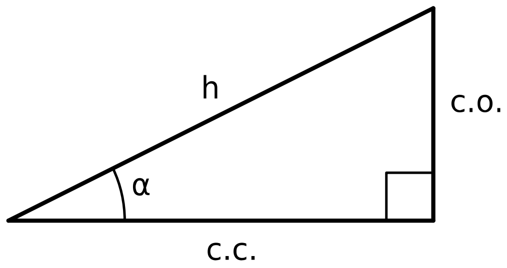
RAONS TRIGONOMÈTRIQUES D'UN ANGLE AGUT



h: hipotenusa

c.o.: catet oposat a l'angle

c.c.: catet contigu a l'angle

Sinus d'un angle

$$\sin\alpha = \frac{c.o.}{h}$$

Cosecant d'un angle

$$\csc\alpha = \frac{h}{c.o.}$$

Cosinus d'un angle

$$\cos\alpha = \frac{c.c.}{h}$$

Secant d'un angle

$$\sec\alpha = \frac{h}{c.c.}$$

Tangent d'un angle

$$\tan\alpha = \frac{c.o.}{c.c.}$$

Cotangent d'un angle

$$\cot\alpha = \frac{c.c.}{c.o.}$$

RELACIONS ENTRE LES RAONS TRIGONOMÈTRIQUES D'UN MATEIX ANGLE

De les definicions de les raons trigonomètriques es dedueixen les següents relacions:

$$\csc\alpha = \frac{1}{\sin\alpha} \qquad \sec\alpha = \frac{1}{\cos\alpha} \qquad \cot\alpha = \frac{1}{\tan\alpha}$$

$$\tan\alpha = \frac{\sin\alpha}{\cos\alpha} \qquad \cot\alpha = \frac{\cos\alpha}{\sin\alpha}$$

I si tenim en compte, que a un triangle rectangle es compleix el teorema de Pitàgores:

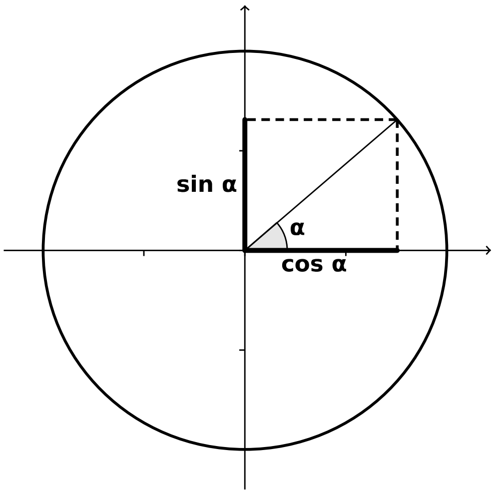
$$\sin^2\alpha + \cos^2\alpha = 1$$

$$\tan^2\alpha + 1 = \frac{1}{\cos^2\alpha} \qquad \tan^2\alpha + 1 = \sec^2\alpha$$

$$\frac{1}{\tan^2\alpha} + 1 = \frac{1}{\sin^2\alpha} \qquad \cot^2\alpha + 1 = \csc^2\alpha$$

RAONS TRIGONOMÈTRIQUES D'UN ANGLE QUALSEVOL

Circumferència goniomètrica: Circumferència de radi 1



$$\tan\alpha = \frac{\sin\alpha}{\cos\alpha}$$

$$\cot\alpha = \frac{\cos\alpha}{\sin\alpha}$$

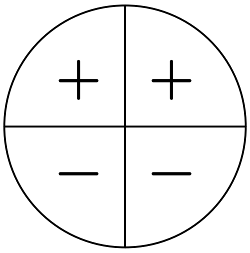
$$\csc\alpha = \frac{1}{\sin\alpha}$$

$$\sec\alpha = \frac{1}{\cos\alpha}$$

SIGNE DE LES RAONS TRIGONOMÈTRIQUES

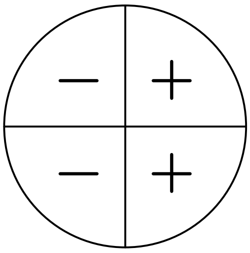
$\sin\alpha$

$\csc\alpha$



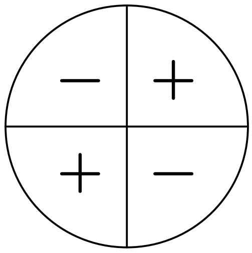
$\cos\alpha$

$\sec\alpha$



$\tan\alpha$

$\cot\alpha$



RECORREGUT DE LES FUNCIONS TRIGONOMÈTRIQUES

$\sin\alpha \in [-1, 1]$ $\quad$ $\cos\alpha \in [-1, 1]$ $\quad$ $\tan\alpha \in \mathbb{R}$

$\csc\alpha \notin (-1, 1)$ $\quad$ $\sec\alpha \notin (-1, 1)$ $\quad$ $\cot\alpha \in \mathbb{R}$

Segons el quadrant:

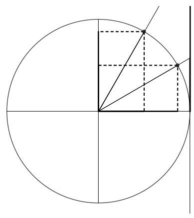
Quadrant I $0° < \alpha < 90°$	**Quadrant II** $90° < \alpha < 180°$	**Quadrant III** $180° < \alpha < 270°$	**Quadrant IV** $270° < \alpha < 360°$
$0 < \sin\alpha < 1$	$0 < \sin\alpha < 1$	$-1 < \sin\alpha < 0$	$-1 < \sin\alpha < 0$
$0 < \cos\alpha < 1$	$-1 < \cos\alpha < 0$	$-1 < \cos\alpha < 0$	$0 < \cos\alpha < 1$
$\tan\alpha > 0$	$\tan\alpha < 0$	$\tan\alpha > 0$	$\tan\alpha < 0$
$\csc\alpha > 1$	$\csc\alpha > 1$	$\csc\alpha < -1$	$\csc\alpha < -1$
$\sec\alpha > 1$	$\sec\alpha < -1$	$\sec\alpha < -1$	$\sec\alpha > 1$
$\cot\alpha > 0$	$\cot\alpha < 0$	$\cot\alpha > 0$	$\cot\alpha < 0$

RELACIONS ENTRE LES RAONS TRIGONOMÈTRIQUES DE DIFERENTS ANGLES

S'utilitzen principalment per a relacionar les raons trigonomètriques d'un angle qualsevol i un del primer quadrant, però són vàlides encara que cap dels angles pertanyi al primer quadrant.

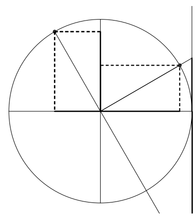
α i $90°-\alpha$
(Angles complementaris)

$$\sin(90°-\alpha) = \cos\alpha$$
$$\cos(90°-\alpha) = \sin\alpha$$
$$\tan(90°-\alpha) = \frac{1}{\tan\alpha}$$



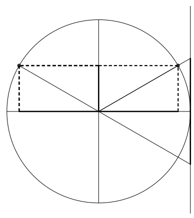
α i $90°+\alpha$

$$\sin(90°+\alpha) = \cos\alpha$$
$$\cos(90°+\alpha) = -\sin\alpha$$
$$\tan(90°+\alpha) = -\frac{1}{\tan\alpha}$$



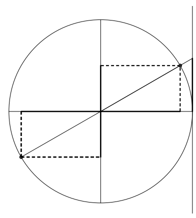
α i $180°-\alpha$
(Angles suplementaris)

$$\sin(180°-\alpha) = \sin\alpha$$
$$\cos(180°-\alpha) = -\cos\alpha$$
$$\tan(180°-\alpha) = -\tan\alpha$$



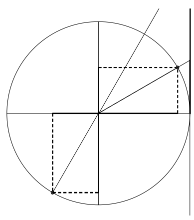
α i $180°+\alpha$
(Angles diametralment oposats)

$$\sin(180°+\alpha) = -\sin\alpha$$
$$\cos(180°+\alpha) = -\cos\alpha$$
$$\tan(180°+\alpha) = \tan\alpha$$



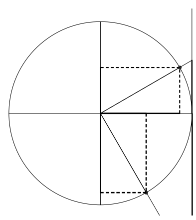
α i $270°-\alpha$

$$\sin(270°-\alpha) = -\cos\alpha$$
$$\cos(270°-\alpha) = -\sin\alpha$$
$$\tan(270°-\alpha) = \frac{1}{\tan\alpha}$$



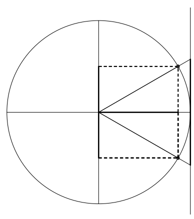
α i $270°+\alpha$

$$\sin(270°+\alpha) = -\cos\alpha$$
$$\cos(270°+\alpha) = \sin\alpha$$
$$\tan(270°+\alpha) = -\frac{1}{\tan\alpha}$$



α i $-\alpha$
(Angles oposats)

$$\sin(-\alpha) = -\sin\alpha$$
$$\cos(-\alpha) = \cos\alpha$$
$$\tan(-\alpha) = -\tan\alpha$$



RAONS TRIGONOMÈTRIQUES EXACTES DE ALGUNS ANGLES

α (°)	α (rad)	$\sin\alpha$	$\cos\alpha$	$\tan\alpha$
0°	0	0	1	0
15°	$\frac{\pi}{12}$	$\frac{\sqrt{6}-\sqrt{2}}{4}$	$\frac{\sqrt{6}+\sqrt{2}}{4}$	$2-\sqrt{3}$
30°	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$
45°	$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
60°	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
75°	$\frac{5\pi}{12}$	$\frac{\sqrt{6}+\sqrt{2}}{4}$	$\frac{\sqrt{6}-\sqrt{2}}{4}$	$2+\sqrt{3}$
90°	$\frac{\pi}{2}$	1	0	∞

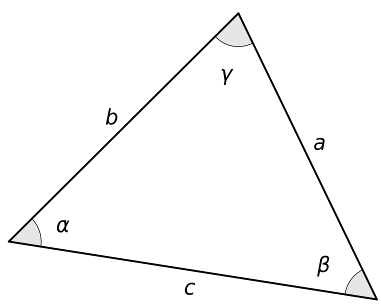
RAONS TRIGONOMÈTRIQUES DE:

LA SUMA DE DOS ANGLES	L'ANGLE DOBLE	L'ANGLE MEITAT
$\sin(\alpha\pm\beta) = \sin\alpha\cdot\cos\beta \pm \cos\alpha\cdot\sin\beta$	$\sin(2\alpha) = 2\cdot\sin\alpha\cdot\cos\alpha$	$\sin\left(\frac{A}{2}\right) = \pm\sqrt{\frac{1-\cos A}{2}}$
$\cos(\alpha\pm\beta) = \cos\alpha\cdot\cos\beta \mp \sin\alpha\cdot\sin\beta$	$\cos(2\alpha) = \cos^2\alpha - \sin^2\alpha$	$\cos\left(\frac{A}{2}\right) = \pm\sqrt{\frac{1+\cos A}{2}}$
$\tan(\alpha\pm\beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\cdot\tan\beta}$	$\tan(2\alpha) = \frac{2\cdot\tan\alpha}{1-\tan^2\alpha}$	$\tan\left(\frac{A}{2}\right) = \pm\sqrt{\frac{1-\cos A}{1+\cos A}}$

TEOREMES DEL SINUS I DEL COSINUS

Teorema del sinus

$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$$



Teorema del cosinus

$$a^2 = b^2+c^2-2bc\cdot\cos\alpha$$

$$b^2 = a^2+c^2-2ac\cdot\cos\beta$$

$$c^2 = a^2+b^2-2ab\cdot\cos\gamma$$